AMAPS Calendar of Lessons
Algebra 2 & Trigonometry
Term One – MR21

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
01	A2.N3	Lesson #1 Aim: How do we perform operations with polynomial expressions containing rational coefficients? Students will be able to 1. add and subtract polynomials with rational coefficients 2. multiply and divide monomials with exponents and rational coefficients 3. simplify parenthetical expressions (nested groupings) 4. multiply polynomials with integral and rational coefficients 5. explain the procedures used to add, subtract, multiply and divide monomials and polynomials Writing Exercise: How is the procedure for multiplying a pair of binomials (FOIL) similar to using the distributive law twice?
02	A2.N3	Lesson #2 Aim: How do we divide polynomials? Students will be able to 1. divide a polynomial by a polynomial including polynomials with rational coefficients; with and without remainders 2. apply the operations of multiplication and addition of polynomials to check quotients Writing Exercise: How is the division of a polynomial by a binomial like the process of long division of two numbers?
03	Review	Lesson #3 Aim: How do we solve first degree equations and inequalities? Students will be able to 1. apply the postulates of equality to solve first degree equations algebraically 2. apply the postulates of inequality to solve first degree inequalities algebraically 3. solve first degree equations and inequalities graphically 4. graphically justify the solution of each linear equation and inequality found algebraically Writing Exercise: Compare the axioms that allow us to solve an equation to those that govern the solution of an inequality.

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04	Review	Lesson #4 Aim: How do we solve and graph compound linear inequalities involving the conjunction and disjunction? Students will be able to 1.graph inequalities on a number line 2. solve and graph conjunctions and disjunctions of two inequalities 3. apply graphing compound inequalities to problems involving the conjunction or the disjunction Writing Exercise: Often the result of a disjunction is a set that has more elements than the set of a conjunction. How is it possible for an "and" situation result in fewer elements than and "or" situation?
05	A2.A46	Lesson #5 Aim: How do we graph absolute value relations and functions? Students will be able to 1. explain how to use the graphing calculator to graph absolute value relations and functions 2. determine the appropriate window for each graph 3. use the graphing calculator to graph absolute value relations and functions 4. apply the transformations f(x+a), f(x) +a, -f(x), and af(x) to the absolute value function Writing Exercise: Using the graphing calculator explore the significance of the coefficient "a" in determining the shape of the graph of y=a|x|.
06	A2.A1	Lesson #6 Aim: How do we solve linear equations in one variable involving absolute values? Students will be able to 1. state the definition of the absolute value of x 2. apply the definition of absolute value to solve linear equations involving absolute values 3. graph solutions to linear absolute value equations on the number line 4. verify solutions to absolute value equations 5. solve verbal problems resulting in linear equations involving absolute value Writing Exercise: The absolute value function is a piecewise function because it has one definition for negative x-values and another definition for positive x-values. In a few sentences explain how this leads to the "derived equations" used in the solution of absolute value equations.

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07	A2.A1	Lesson #7 Aim: How do we solve linear absolute value inequalities involving one variable? Students will be able to 1. solve inequalities of the form $\|x\| < k$ and $\|x\| > k$ 2. solve inequalities of the form $\left\|x \pm a\right\| < k \text{ and } \left\|x \pm a\right\| > k$ 3. graph solutions to linear absolute value inequalities on the number line 4. verify solutions to absolute value inequalities 5. solve verbal problems resulting in linear inequalities involving absolute value Writing Exercise: Describe the circumstances under which the graph of the solution set to an absolute value inequality will be made up of two disjoint sets. Describe the circumstances under which it will be a continuous interval.
08	A2.A7	Lesson #8 Aim: How do we factor polynomials? Students will be able to 1. recognize when to factor out a greatest common factor 2. factor by extracting the greatest common factor 3. identify and factor quadratic trinomials, where $a \geq 1$ 4. factor quadratic trinomials that are perfect squares 5. justify the procedures used to factor given polynomial expressions Writing Exercise: Why is factoring a polynomial like a question from the quiz show <u>Jeopardy</u>?
09	A2.A7	Lesson #9 Aim: How do we factor the difference of two perfect squares and factor polynomials completely? Students will be able to 1. recognize and factor the difference of two perfect squares 2. compare factoring the difference of two perfect squares to factoring a trinomial 3. explain what is meant by factoring completely 4. factor polynomials completely 5. justify the procedures used to factor polynomials completely 6. explain why it is more efficient to factor completely by first extracting the GCF 7. factor cubic expressions of the form $a^3 - b^3$ or $a^3 + b^3$ (enrichment only) Writing Exercise: How is factoring a polynomial <u>completely</u> like reducing a fraction to lowest terms?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
09	Review Lesson	Lesson #9 Aim: How do we solve quadratic equations by factoring? Students will be able to 1. justify and explain the multiplication property of zero 2. explain what is meant by the standard form of a quadratic equation 3. transform a quadratic equation into standard form 4. factor the resulting quadratic expression 5. apply the multiplication property of zero to solving quadratic equations 6. check the answers in the original equation Writing Exercise: When the product of two factors is zero we can make a conclusion about one or more of the factors. What is this conclusion and what property allows us to make the conclusion?
10	Review Lesson	Lesson # 10 Aim: How do we graph the parabola $y = ax^2 + bx + c$? Students will be able to 1. create a table of values and graph a parabola 2. explain the effect of a, b, and c on the graph of the parabola 3. identify the key elements of a parabola (axis of symmetry, turning point, intercepts, opening up/down) 4. investigate and discover the effects of the transformations $f(x+a)$, $f(x) + a$, $af(x)$, $f(-x)$ and $–f(x)$ on the graph of a parabola 5. produce a complete graph of a parabola using a graphing calculator 6. use the graphing calculator to identify the roots of a quadratic equation Writing Exercise: Video games use life-like graphics to model 3-D effects on the 2-D computer monitor. Video golf games show the path of a golf ball that is hit by each player. Describe the shape of the path of the ball as shown on the screen and speculate on the type of equation(s) the programmer needed to use in order to produce this graphic.
11	A2.A4	Lesson #11 Aim: How do we solve and graph a quadratic inequality algebraically? Students will be able to 1. transform a quadratic inequality into standard form 2. solve a quadratic inequality algebraically and graph the solution on a number line 3. write the solution to a quadratic inequality as a compound inequality Writing Exercise: How can the graphing calculator be used to verify the solution set of its related quadratic inequality?

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12	A2.A4	Lesson #12 Aim: How can we use the graph of a parabola to solve quadratic inequalities in two variables? Students will be able to 1. identify the key elements (axis of symmetry, turning point, intercepts, opening up/down) of a parabola 2. apply transformations to the graph of a parabola 3. produce a complete graph of a parabola using a graphing calculator 4. use the graphing calculator to calculate the roots of a quadratic equation 5. solve a quadratic inequality in two variables graphically 6. write the solution to a quadratic inequality as a compound inequality Writing Exercise: In class, we plot 6 or 8 points to define the complete shape of a parabola. How do we determine what x-values to use in order to create a complete graph of the parabola?
13	A2.A4	Lesson #13 Aim: How do we solve more complex quadratic inequalities? Students will be able to 1. transform a quadratic inequality into standard form 2. solve quadratic inequalities 3. graph the solution set of a quadratic inequality on the number line 4. apply graphing to the conjunction and the disjunction 5. graph the related parabola to identify the solution to a quadratic inequality Writing Exercise: A produce supply company provides fresh fruit and vegetables to local grocery stores and restaurants. For the supply store to be profitable it must have enough produce on hand to meet customer orders. At the same time since fruits and vegetables spoil easily it cannot overstock its warehouse. Consider the solution of a quadratic inequality; compare the graph of its solution set to the profit situation of the produce supply company.
14	A2.A13 A2.N2	Lesson #14 Aim: How do we simplify radicals? Students will be able to 1. simplify radicals with numerical indices of 2 or more 2. simplify radicals involving monomial radicands 3. explain the procedure for simplifying radical expressions 4. explain how to determine when a radical is in simplest form Writing Exercise: The *irrational numbers* were known to the Pythagoreans, but were largely ignored by them. (The Pythagoreans were a group of mathematicians who lived around the time of Pythagoras.) Use the resources of the internet or your local library to investigate why the irrational numbers were ignored. How does the answer to this question explain the name "irrational numbers?"

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15	A2.A14 A2.N2 A2.N4	Lesson #15 Aim: How do we add and subtract radicals? Students will be able 1. add and subtract like radicals with numerical or monomial radicands 2. add and subtract unlike radicals with numerical or monomial radicands 3. express the sum or difference of radicals in simplest form 4. explain how to combine radicals Writing Exercise: Why is x^6 a perfect square monomial and x^9 not a perfect square monomial?
16	A2.A14 A2.N2 A2.N4 A2.N5	Lesson #16 Aim: How do we multiply and divide radicals? Students will be able to 1. multiply radical expressions with numerical or monomial radicands 2. divide radical expressions with numerical or monomial radicands 3. express the products and quotients of radicals in simplest form 4. express fractions with irrational monomial denominators as equivalent fractions with rational denominators Writing Exercise: Compare operations with radicals to operations with monomials. In what ways are they the same or different?
17	A2.A15 A2.N5	Lesson #17 Aim: How do we rationalize a fraction with a radical denominator (monomial or binomial)? Students will be able to 1. express fractions with irrational monomial denominators as equivalent fractions with rational denominators 2. define binomial surd and conjugate 3. express fractions with irrational binomial denominators as equivalent fractions with rational denominators 4. express results in simplest form Writing Exercise: Why is it the convention to rationalize denominators?
18	A2.A24	Lesson # 18 Aim: How do we complete the square? Students will be able to 1. identify a perfect square trinomial 2. factor a perfect square trinomial 3. express a perfect square trinomial as the square of a binomial 4. state the relationship between the coefficient of the middle term and the constant term of a perfect square trinomial 5. determine the constant term that needs to be added to a binomial or trinomial to make it a perfect square trinomial 6. solve quadratic equations by completing the square Writing Exercise: Why do we refer to numbers like: 16, 25 and 100 as perfect “square” numbers? Explain whether or not your answer to this first question also applies to perfect square trinomials. Give an example of a perfect square trinomial.

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19	A2.A24 A2.A25	Lesson #19 Aim: How do we apply the quadratic formula to solve quadratic equations with rational roots? Students will be able to 1. apply the method of completing the square to discover the quadratic formula 2. state the quadratic formula 3. apply the quadratic formula to solve quadratic equations 4. express rational roots in simplest form 5. solve verbal problems that result in quadratic equations 6. use the zero finder of a graphing calculator to find the roots of a quadratic equation with rational roots Writing Exercise: How is the quadratic formula related to the process of completing the square?
20	A2.A25	Lesson #20 Aim: How do we apply the quadratic formula to solve quadratic equations with irrational roots? Students will be able to 1. state the quadratic formula 2. apply the quadratic formula to solve quadratic equations 3. express irrational roots in simplest radical form 4. approximate irrational roots in decimal form to a specified degree of accuracy 5. solve verbal problems that result in quadratic equations 6. use the zero finder of a graphing calculator to approximate the roots of a quadratic equation with irrational roots Writing Exercise: How can the graphing calculator be used to verify the solution set of the related quadratic function?
21		Lesson #21 Aim: How do we apply the quadratic formula to solve verbal problems? Students will be able to 1. state the quadratic formula 2. create an appropriate quadratic equation that can be used to solve the verbal problem 3. use the quadratic formula to identify possible solutions (roots) 4. use the zero finder of a graphing calculator to verify the roots of a quadratic equation 5. verify each solution in the words of the problem Writing Exercise: Tronty's answer to his real world quadratic verbal problem gave him two roots. He rejected the negative root. Deepak solved the same problem and kept BOTH roots as valid solutions to the verbal problem. Describe real-world situation in which each student would be correct.

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22	A2.N6 A2.N7	Lesson #22 Aim: What are properties of complex numbers? Students will be able to 1. define a vector, an imaginary number, and a complex number 2. simplify powers of *i* 3. differentiate between complex and imaginary numbers 4. express imaginary and complex numbers in terms of i 5. plot points on the complex number plane Writing Exercise: The history of imaginary numbers is much like that of the irrational number. Use the resources of the Internet or the local library to find out about the early discovery of imaginary numbers. Why do you think that they were called "imaginary numbers?"
23	A2.N8 A2.N9	Lesson #23 Aim: How do we add and subtract complex numbers? Students will be able to 1. add and subtract complex numbers algebraically and express answers in simplest a+b*i* form 2. add and subtract complex numbers graphically and express answers in simplest a+b*i* form 3. find the additive inverse of complex numbers Writing Exercise: Complex numbers are a new group of numbers yet they behave like variables or radicals under binary operations. Describe these similarities.
24	A2.N8 A2.N9	Lesson #24 Aim: How do we multiply complex numbers? Students will be able to 1. multiply and simplify expressions that involve complex numbers 2. define a pair of conjugates 3. write the conjugate of a given complex number Writing Exercise: What are conjugate pairs? What makes them special? How are they related to the conjugate pairs we found we rationalizing the denominator of a fraction with an irrational binomial denominator?
25	A2.N8 A2.N9	Lesson #25 Aim: How do we divide complex numbers? Students will be able to 1. write the conjugate of a given complex number 2. find the quotient of two complex numbers and express the result with a real denominator 3. express the multiplicative inverse of a complex number in standard a+bi form Writing Exercise: How do conjugate pairs help to simplify fractions with complex denominators?

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26	A2.A2	Lesson #26 Aim: How do we find complex roots of a quadratic equation using the quadratic formula? Students will be able to 1. solve a quadratic equation that leads to complex roots 2. express complex roots in a + b*i* form Writing Exercise: Explain why some quadratic equations have imaginary or complex roots.
27	A2.A2	Lesson 27: Aim: How do we use the discriminant to determine the nature of the roots of a quadratic equation? Students will be able to 1. define the discriminant 2. determine the value of the discriminant 3. determine the nature of the roots using the discriminant Writing Exercise: Justify the use of the discriminant as the quickest way to determine the nature of the roots of a quadratic equation.
28	A2.A20 A2.A21	Lesson #28 Aim: How do we find the sum and product of the roots of a quadratic equation? Students will be able to 1. find the sum and product of the roots of a quadratic equation 2. find the value of an unknown coefficient of a quadratic equation given one root of the equation 3. check the solutions to quadratic equations using the sum and product relationships 4. write a quadratic equation given the sum and product of its roots 5. write a quadratic equation when both roots are known Writing Exercises: 1. What is the relationship between the roots of a quadratic equation and the coefficients of the quadratic equation? How does writing the equation in standard form impact on this relationship? 2. We are often asked to find the equation given its roots. How does the relationship between the sum and product of the roots make it easier to find the equation when the roots are complex?

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29	Review Lesson	Lesson #29 Aim: How do we solve quadratic-linear systems of equations using the graphing calculator? . Students will be able to 1. explain what is meant by a system of equations and by the solution to a system of equations 2. explain how to use the graphing calculator to graph each equation 3. explain how to use the graphing calculator to solve systems of equations 4. determine the graphing window for a system of equations 5. identify the graphs from their equations 6. solve systems of equations using the graphing calculator 7. check the solution to the system of equations Writing Exercise: Describe the significance of choosing the correct graphing window for a system of equations whose solution you are seeking. Give an example in which the window parameters are critical.
30	A2.A3	Lesson #30 Aim: How do we solve quadratic-linear systems of equations algebraically? Note: This includes rational equations that can be written as linear equations with restricted domain, which, if not carefully considered might produce extraneous roots for the system. i.e. $\frac{y}{x}=1 \text{ and } y=x^2-x$. Students will be able to 1. explain how to solve for one variable in terms of the other 2. explain how to substitute one equation into the other to create one equation in one variable 3. algebraically solve the system of equations for all possible solutions 4. algebraically check the solutions to the system of equations 5. graphically verify the solution of a quadratic-linear system found algebraically Writing Exercise: A quadratic-linear system can have one, two or no solutions. By referring to the graphs of a quadratic-linear system, explain how this is possible.

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31	A2.A16	Lesson #31 Aim: How do we reduce rational expressions? Students will be able to 1. explain the circumstances under which a rational expression is undefined 2. find the value(s) that make a rational expression undefined 3. reduce rational expressions to lowest terms 4. explain the procedure used for reducing fractions to lowest terms 5. explain the circumstances under which a rational expression is in lowest terms Writing Exercises: 1. Under what circumstances is the expression $\frac{x+5}{x+5}$ not equal to 1? Explain what impact this might have on the reduction of a rational expression. 2. Rudy simplified the following expression $\frac{2x+7}{2x+6}$ and obtained the answer $\frac{7}{6}$. Discuss what he did to get his answer and determine whether or not his method is valid.
32	A2.A16	Lesson #32 Aim: How do we multiply and divide rational expressions? Students will be able to 1. multiply rational expressions and express the product in lowest terms 2. divide rational expressions and express the quotient in lowest terms 3. compare and contrast the procedures used to multiply and divide rational expressions Writing Exercise: If the expression $\frac{x+5}{5+x}$ is equal to one, explain why the expression $\frac{x-5}{5-x}$ is not equal to one. What is the value of this expression and how can this be used to reduce a rational expression?
33	A2.A16	Lesson #33 Aim: How do we add and subtract rational expressions with like denominators or unlike monomial denominators? Students will be able to 1. add and subtract rational expressions with like denominators, and reduce answers where applicable 2. explain how to find the least common denominator 3. add and subtract rational expressions with unlike monomial denominators, and reduce answers where applicable 4. explain the procedure used to add and subtract rational expressions Writing Exercise: How are adding or subtracting rational expressions with like denominators similar to combining like terms?

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34	A2.A16	Lesson #34 Aim: How do we add and subtract rational expressions with unlike polynomial denominators? Students will be able to 1. explain how to find the least common denominator 2. add and subtract rational expressions with unlike denominators 3. reduce answers when applicable 4. explain the procedure used to add and subtract rational expressions with unlike denominators Writing Exercise: Why is it inaccurate to simply add the numerators of two fractions with unlike denominators?
35	A2.A17	Lesson #35 Aim: How do we reduce complex fractions? Students will be able to 1. define complex fraction 2. simplify complex fractions 3. reduce fractions whose numerator and denominator have factors that are additive inverses 4. compare and explain procedures used to simply complex fractions Writing Exercise: How can the *multiplicative property of one* be used to simplify a complex fraction?
36	A2.A23	Lesson #36 Aim: How do we solve rational equations? Students will be able to 1. determine the appropriate LCM 2. apply finding the LCM to solving rational equations 3. explain what is meant by an extraneous root 4. conjecture the circumstances under which an equation may have an extraneous root 5. check answers to determine if roots are extraneous 6. compare the procedure used to simply complex fractions with the procedure used to solve rational equations 7. contrast the process of combining algebraic fractions with solving rational equations Writing Exercise: Compare and contrast the process of combining algebraic fractions with solving fractional equations.
37	A2.A4 A2.A23	Lesson #37 Aim: How do we solve rational inequalities? Students will be able to 1. identify a rational inequality 2. determine the zeros of the numerator and denominator 3. use the zeros to perform the sign analysis on both the numerator and the denominator 4. determine the intervals in which the numerator and denominator have positive or negative values 5. create the solution set Writing Exercise: How is the multiplication of both sides of an inequality different from multiplication of both sides of an equation?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
38	A2.A8 A2.A9 A2.A10 A2.A11 A2.N1	Lesson #38 Aim: How do we evaluate expressions involving negative and rational exponents? Students will be able to 1. define what is meant by a rational exponent 2. simplify expressions involving rational exponents with and without a calculator 3. define what is meant by a negative exponent 4. simplify expressions involving negative exponents with and without a calculator 5. evaluate expressions involving rational exponents 6. write algebraic expressions using negative or rational exponents Writing Exercise: Explain the difference between the meaning of $x^{\frac{1}{3}}$ and x^{-3}. Give an example that supports your explanation.
39	A2.A22	Lesson #39 Aim: How do we find the solution set for radical equations? Students will be able to 1. conjecture and apply the procedure for solving radical equations of index 2 2. check solutions to determine any extraneous roots 3. solve radical equations involving two radicals 4. conjecture and explain the procedure used to solve radical equations with index 3 5. solve radical equations with index 3 6. explain why solutions must be checked in the original equation Writing Exercise: Sometimes the solution to a radical equation produces an extraneous root. Describe what an extraneous root is and tell what it is about the process of solving radical equations that causes the extraneous root to occur.
40		Lesson #40 Aim: How do we find the solution set of an equation with fractional exponents? Students will be able to 1. solve equations with fractional exponents 2. verify solutions to equations with fractional exponents Writing Exercise: How does the process used to solve an equation with fractional exponents produce extraneous roots? How do we guard against claiming that we have a root when it really is an extraneous root?

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41	A2.A37 A2.A39 A2.A51 A2.A52	Lesson #41 Aim: What are relations? Students will be able to 1. state the definition of a relation 2. identify the domain and range of a relation expressed as an equation, table or graph 3. use a variety of mapping techniques to demonstrate a relation 4. create and describe a relation using real world situations Writing Exercise: The words input and output are often used to describe the domain and range of a relation. Explain why these words make sense and justify why a relation is sometimes compare to a "machine."
42	A2.A37 A2.A38 A2.A39 A2.A51 A2.A43 A2.A52	Lesson #42 Aim: What are functions? Students will be able to 1. state the definition a function 2. explain the difference between a relation and a function 3. identify the domain and range of a function expressed as an equation, table or graph 4. determine if a relation is a function using the definition 5. identify relations and function by examining the graph 6. define functions that are one-to-one and functions that are onto 7. determine if a function is one-to-one, onto, or both Writing Exercise: Explain the following analogy: *poodle is to dog as function is to relation*
43	A2.A39 A2.A40 A2.A41 A2.A43	Lesson #43 Aim: How do we use function notation? Students will be able to 1. use function notation to evaluate functions for given values in the domain 2. find the domain and range of a function 3. write functions in functional notation 4. evaluate functions using function notation given a numerical or an algebraic input 5. determine if a function is one-to-one, onto, or both Writing Exercise: If function notation is just another way to write "y," what is the reason for inventing this notation?

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44	A2.A42	Lesson #44 Aim: What is composition of functions? Students will be able to 1. define composition and identify its symbol "$\circ$" 2. state the order of operations in composition of functions 3. express composition using appropriate composition notation: $f \circ g(x)$ or $f(g(x))$ 4. apply composition of functions to numerical and algebraic examples Writing Exercise: Addition is a binary operation. Many textbooks refer to composition as a binary operation. Explain this thinking.
45	A2.A44 A2.A45	Lesson #45 Aim: How do we find the inverse of a given relation? Students will be able to 1. state and write the meaning of an inverse relation 2. determine if the inverse of a function is also a function 3. form the inverse of given relations or functions 4. apply composition of functions to verify that two functions are inverses of each other 5. draw the graph of the inverse of a relation or function Writing Exercise: Each function has a domain and a range. When the inverse of a function is found, the domain and range are reversed. What transformation of the plane is created when the x and the y values are reversed? What does this mean about the graphical relationship between a function and its inverse?
46	A2.A6 A2.A12 A2.A53	Lesson #46 Aim: What is an exponential function? Students will be able to 1. define what is meant by the exponential function 2. state an approximate value of the irrational number e 3. sketch the graph of $y = a^x$ where $a > 0$, a is not equal to 1, including functions where: $y = e^x$ 4. compare exponential graphs with linear or quadratic graphs 5. model exponential functions using a graphing calculator 6. solve problems which result in an exponential function Writing Exercise: 1. Compare π to e. 2. Radioactive iodine is a by-product of some types of nuclear reactors. Its half-life is 60 days. Suppose a nuclear accident happens and a fixed amount of the radioactive iodine is given off. Discuss the amount of radioactive iodine that will be present during the next 180 days using the half-life as intervals for your discussion. What graphical model will best project the amount after 320 days? Writing Project: Research the history of e and describe real-world applications in which it is found.

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47	A2.A12 A2.A54	Lesson #47 Aim: What is the inverse of the exponential function? Students will be able to 1. define what is meant by the logarithmic function 2. explain that the logarithmic function is the inverse of the exponential function 3. apply inverses to discover the relationship between $y = \log_a x$ and $y = a^x$, and between $y = \ln x$ and $y = e^x$ 4. sketch the graph of $y = \log_a x$ using the graph of $y = a^x$ 5. sketch the graph of $y = \ln x$ using the graph of $y = e^x$ 6. compare properties of the exponential and the logarithmic graphs 7. apply the inverse relationship of logarithms and exponents to simplify expressions Writing Exercise: When we form the inverse of a function we interchange the x and y. Form the inverse of $y=a^x$ and describe the difficulty in solving for y in this example. How do we resolve this difficulty?
48	A2.A12 A2.A18 A2.A19 A2.A28	Lesson #48 Aim: How do we find the $\log_b a$? Students will be able to 1. convert from exponential to logarithmic form and vice versa, including $\log_b a$ and ln a 2. evaluate without the use of calculator, $\log_b a$ (where a is a power of b) 3. solve logarithmic equations by converting to exponential form 4. use a calculator to find a. the log of a number with base of 10 b. the natural log of a number 5. use a calculator to find a number given its logarithm Writing Exercise: The technique of logarithms was invented before the invention of the hand-held calculator. What advantage did logarithms provide to a world without calculators?
49	A2.A19	Lesson #49 Aim: How do we use logarithms to find values of products and quotients? Students will be able to 1. state the rules for finding the log of a product and the log of a quotient 2. state the rules for finding the ln of a product and the ln of a quotient 3. write a log equation to find the product and/or the quotient of two literal factors 4. apply the laws of logarithms concerning products and quotients 5. apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms Writing Exercise: Explain why logarithms can be helpful in finding products and quotients but are not helpful in finding sums and differences.

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50	A2.A19	Lesson #50 Aim: How do we use logarithms for raising a number to a power or finding roots of numbers? Students will be able to 1. state the rule for finding the log of a number raised to a power 2. apply the laws of logarithms to an expression involving integral and rational exponents 3. use laws of logarithms to write a logarithmic equation for a given literal expression 4. apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms Writing Exercise: If logarithms were useful in a time without calculators, why then do we still need to study logarithms?
51	A2.A6 A2.A27	Lesson #51 Aim: How do we solve exponential equations? Students will be able to 1. define an exponential equation 2. solve exponential equations with common bases 3. verify solutions to exponential equations 4. state the principle used in solving exponential equations 5. apply exponential equations in the solution of verbal problems 6. evaluate using the graphing calculator Writing Exercise: Explain how the graph of an exponential function can be used to solve specific exponential equations.
52	A2.A6 A2.A27	Lesson #52 Aim: How do we solve exponential and logarithmic equations? Students will be able to 1. use logarithms to solve exponential equations without common bases 2. solve logarithmic equations 3. apply exponential equations in the solution of verbal problems Writing Exercise: Describe three ways that $2^x=8$ can be solved.
53	A2.A.27	Lesson#53 Aim: How do we solve verbal problems involving exponential growth or decay? Students will be able to 1. apply their knowledge of solving exponential and logarithmic equations to verbal problems involving: a. finance including interest (simple and compound) and investments b. exponential growth c. half-life 2. solve verbal problems described above using the graphing calculator Writing Exercise: Jacob has 18 kg of radium. If the half-life of radium is 13 years, how many years will it take until the radium is all “gone”? Explain your answer.

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54	Review Lesson	Lesson #54 Aim: What are the transformations involving reflections? Students will be able to 1. state and write a definition for transformation symmetry 2. apply transformation definitions of vertical and horizontal line symmetry to reflections over the x-axis, the y-axis and the line y=x 3. use a straightedge to draw lines of symmetry 4. determine the properties preserved under a line reflection 5. apply the above transformations to coordinate geometry 6. define and reflect points, lines and triangles through the origin 7. show the reflection through a point is a half-rotation, not a flip 8. list the properties preserved under a point reflection Writing Exercise: Give an example of a transformation in nature.
55	A2.A46	Lesson #55 Aim: What are geometric translations, dilations and rotations? Students will be able to 1. define a translation and a dilation as a transformation 2. apply and write the general notation for any translation as $T_{(a,b)}(x,y) \rightarrow (x+a,y+b)$ 3. apply and write the general notation for any dilations as $D_k(x,y) \rightarrow (kx, ky)$ 4. apply translations and dilations to numerical examples 5. define rotation about a point 6. apply clockwise and counterclockwise rotations 7. state, graph and write which properties are preserved under translations, dilations and rotations Writing Exercise: Explain how is it possible to have a negative constant of dilation?
56	A2.A46	Lesson #56 Aim: How do we perform transformations of the plane on relations and functions? Students will be able to 1. perform transformations of the form $f(x+a)$, $f(x)+a$, $-f(x)$, $f(-x)$, and $af(x)$ on relations and functions 2. explain the effect on the graph of f(x) for each of the above listed transformations 3. write the equations of graphs that were shifted vertically, shifted horizontally or reflected over the x-axis Writing Exercise: Transformations are mappings that assign each point on the plane onto its image according to a rule. Compare each of the transformation rules discussed in class today. Describe how the function and its image are the same and how they are different.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
57	A2.A47 A2.A49 A2.A48	Lesson #57 Aim: How do we graph and write the equation of a circle? Students will be able to 1. write the equation of a circle given a. the coordinates of its center and the length of its radius b. the coordinates of its center and a point on the circle c. the coordinates of two points and the center d. its graph 2. graph a circle with the center at the origin from its equation 3. graph a circle with the center at (h, k) from its equation 4. apply transformations to the graph of a circle 5. graph a circle using the graphing calculator 6. transform the equation of circle given in general form to center-radius form by completing the square 7. identify the coordinates of the center and the length of the radius of a circle from a graph or an equation Writing Exercise: For circles with center at (h, k), explain how the equation reflects the fact that the circle has been translated from the origin.
58	A2.A5	Lesson #58 Aim: What is direct and inverse variation? Student will be able to 1. define direct variation 2. define inverse variation 3. identify situations involving direct and inverse variation 4. solve algebraic problems related to direct and inverse variation 5. relate direct variation to the graph of a straight line 6. graph inverse variation and identify the graph as an equilateral hyperbola Writing Exercise: Describe a real-life circumstance that illustrates the concept of direct variation. Amend your idea to illustrate the concept of inverse variation.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
59	A2.A26 A2.A50	Lesson #59 Aim: How do we find the roots of polynomial equations of higher degree by factoring and by applying the quadratic formula? Students will be able to 1. define the degree of a polynomial equation 2. factor polynomial expressions of degree ≥ 3 3. identify polynomials that are written in 'quadratic form' 4. state and apply the quadratic formula 5. express irrational solutions in simplest radical form 6. graphically identify (estimate) x-intercepts as solution of a polynomial equation 7. graphically identify (estimate) the x-coordinate of the point of intersection of a system of polynomial equations as a solution of that system Writing Exercise: Describe why the exponents of each variable term in an polynomial expression in quadratic form can be written in the form of 2n, n and 0 where n is an integer.

AMAPS Calendar of Lessons
Algebra 2 & Trigonometry
Term Two - MR22

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
60	A2.A55	Lesson # 1 Aim: What are the six trigonometric functions of an angle? Students will be able to 1. define sine, cosine and tangent of an acute angle in a right triangle 2. define secant, cosecant, and cotangent as the reciprocal functions of cosine, sine, and tangent respectively 3. express the value of the six trigonometric functions of an angle as ratios of the sides of a right triangle 4. use the appropriate calculator key strokes to find the values of reciprocal functions 5. explain that the product of a function and its reciprocal is one 6. express values of other trigonometric functions when given the value of one of the trigonometric function values 7. determine the value of a trig function using technology Writing Exercise: The word "trigonometry" has Greek roots. Look up these roots and compare the roots to the mathematical definition of trigonometry.
61	A2.A56	Lesson # 2 Aim: What are the properties of the special right triangles? Students will be able to 1. investigate the relationships between the sides of a 30^0-60^0-90^0 triangle and a 45^0-45^0-90^0 triangle 2. given the length of any side of a 30-60-90 triangle or a 45^0-45^0-90^0 triangle, express the exact and approximate lengths of the other two sides 3. find the trigonometric function values using a 30-60-90 triangle 4. find the trigonometric function values using a 45-45-90 triangle 5. apply these special right triangle relationships to evaluate numeric and algebraic expressions involving functions of angles measuring 30^0, 45^0, and 60^0 Writing Exercise: Speculate as to why the special right triangles discussed in today's lesson are so important in the real world.

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Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
62	A2.A61 A2.M1 A2.M2	Lesson #3 Aim: How do we use radians to measure angles? Students will be able to 1. define a radian 2. discover and conjecture the relationship between degree measure and radian measure 3. convert radian measure into degree measure and vice versa 4. evaluate expressions involving trig functions whose angles are given in radians Writing Exercise: What is the degree measure of an angle whose radian measure is equal to 1? Would radian measure be appropriate for very fine measurements? Justify your answer.
63	A2.A61	Lesson #4 Aim: How do we find the length of an arc? Students will be able to 1. investigate and discover the relationship between the radian measure of an angle and the length of its intercepted arc 2. conjecture and apply the formula $S=\theta r$ Writing Exercise: How do degree measure and arc length differ?
64	A2.A55 A2.A58 A2.A59	Lesson #5 Aim: What are co-functions and quotient identities? Students will be able to 1. state that sine and cosine, tangent and cotangent, secant and cosecant are co-functions 2. solve trigonometric equations using the principle "If the co-functions of two acute angles are equal, then the angles are complementary" 3. compare co-function and reciprocal relations 4. discover and apply quotient identities 5. express quotients of trigonometric functions in terms of sine and cosine Writing Exercise: The words cotangent, cosecant and cosine all begin with the prefix "co." Explain a reason for this.
65	A2.A67	Lesson #6 Aim: What are the Pythagorean Identities? Students will be able to 1. investigate, discover, and conjecture the Pythagorean Identities 2. justify the validity of the Pythagorean Identities using special angles 3. simplify trigonometric expressions by substituting previous learned identities 4. explain which identities express relationships between the same angle and which identities express relationships between different angles (i.e., between angles that are complementary) Writing Exercise: Explain why the name Pythagorean Identity is appropriate.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
66	A2.A57 A2.A60 A2.A66	Lesson #7 Aim: How do we define the trigonometric ratios for angles of any size? Students will be able to 1. explain what is meant by an angle drawn in standard position and by a reference angle 2. identify the quadrant in which an angle terminates 3. express sine, cosine, tangent, cotangent, cosecant, secant functions in terms of rectangular coordinates 4. conjecture and justify which functions are positive in each quadrant 5. determine the quadrant of an angle when given a point on the terminal side of the angle 6. find the number of degrees in an angle when given the coordinates of a point on the terminal side using reference angles of 30°, 45° or 60° 7. sketch and label the unit circle and represent angles in standard position 8. identify the line segment of a unit circle that represents each of the six trig functions of an angle 9. use the concept of the unit circle to solve real-world problems involving trigonometric functions 10. determine the value of a trig function using technology Writing Exercise: Explain an advantage of using the unit circle to define the trigonometric ratios?
67	A2.A57 A2.A59 A2.A60 A2.A62	Lesson #8 Aim How do we find functions of angles greater than 90 degrees? Students will be able to 1. explain what is meant by a reference angle 2. use a diagram to determine the sign of the required function in any quadrant 3. relate functions of angles greater than 90° to the same function of an acute angle in quadrant I 4. state the definition of co-terminal angles 5. explain how to find the reference angle for an angle in any quadrant 6. state and apply the procedure used for finding trigonometric functions of any angle expressed in degree or radian measure 7. explain how to determine the appropriate sign when you express the function of an angle in terms of a function of its reference angle 8. find the value of the six trig functions given the coordinates of a point on the terminal side of the angle in standard position Writing Exercise: How do the definitions of the trigonometric ratios in terms of rectangular coordinates extend the meaning of the trigonometric functions beyond the limits of the right triangle?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
68	A2.A56 A2.A57 A2.A59	Lesson #9 Aim: How do we find functions of negative angles and quadrantal angles? Students will be able to 1. explain what is meant by a negative angle 2. relate a negative angle expressed in degree or radian measure to its positive co-terminal angle 3. state what is meant by a quadrantal angle 4. find functions of quadrantal angles using the unit circle 5. verify the values of functions of quadrantal angles using a calculator 6. express functions of negative angles as functions of positive angles 7. use a calculator to check values of trigonometric functions 8. evaluate trigonometric expressions containing quadrantal and negative values 9. find the exact value of functions of quadrantal angles, when the angles are expressed in radians or degrees Writing Exercise: 1. How can an angle be negative? 2. How are quadrantal angles different from all the other angles?
69	A2.A59 A2.A62	Aim: #10 Aim: How do we find the other trigonometric function values given the value of one trigonometric function? Students will be able to 1. conjecture and explain the procedure used to find trigonometric functions of any angle 2. explain the difference between finding the function of an angle and finding the value of the function of an angle 3. compute the other trigonometric function values, when given the value of one of the trigonometric function values 4. use technology to find the function values for angles expressed in radians and angles expressed in degrees and minutes Writing Exercise: How can the relationship between a pair of complementary angles be used to express a function of an angle as a function of an angle that is less than 45°?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
70	A2.A69	Lesson #11 Aim: How do we draw the graphs of $y = \sin x$ and $y = \cos x$? Students will be able to 1. generate and use tables to graph $y = \sin x$ and $y = \cos x$ 2. use the graphing calculator to verify the graphs of $y = \sin x$ and $y = \cos x$ 3. analyze the graphs to find the sine and cosine values of quadrantal angles 4. investigate and explain how both graphs vary in the four quadrants 5. analyze the graphs to conjecture the coordinates of maximum and minimum points 6. conjecture and explain the domain and range of each function 7. investigate the interval for which each function repeats itself *8.* define *amplitude* and *period* 9. compare and contrast the sine and cosine graphs for period, amplitude, coordinates of maximum and minimum points Writing Exercise: Explain the following statement: “The time of sunrise at a particular location for a full year can be represented by a sine curve.”
71	A2.A69 A2.A70 A2.A72	Lesson #12 Aim: How do we sketch the graphs of $y = a\sin bx$ and $y = a\cos bx$? Students will be able to 1. define *amplitude*, *period* and *frequency* 2. state the amplitude, frequency, period, domain and range of $y = a\sin bx$ and $y = a\cos bx$ 3. sketch $y = a\sin bx$ and $y = a\cos bx$ using the five critical points 4. use the graphing calculator to graph $y = a\sin bx$ and $y = a\cos bx$ 5. explore and conjecture the effects on the graph as "a" changes and as “b” changes 6. determine the amplitude, frequency, period, domain, and range from the equation 7. write the trigonometric function that is represented by a given periodic graph Writing Exercise: What do you think is the advantage of expressing angle measures in terms of radians?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
72	A2.A69 A2.A72	Lesson #13 Aim: How do we sketch the graphs of $y = a\sin(bx+d)+c$ and $y = a\cos(bx+d)+c$? Students will be able to 1. find the amplitude, frequency, period, domain and range of $y = a\sin(bx+d)+c$ and $y = a\cos(bx+d)+c$ 2. investigate and conjecture the effects on the graph as b, d, and c change 3. sketch $y = a\sin(bx+d)+c$ and $y = a\cos(bx+d)+c$ using the five critical points 4. use the graphing calculator to graph $y = a\sin(bx+d)+c$ and $y = a\cos(bx+d)+c$ 5. write the equation of a sine or cosine function whose graph has a specified period and amplitude 6. determine phase shift given the graph or equation of a periodic function 7. write the trig function that is represented by a given periodic graph Writing Exercise: How is the periodic nature of the sine and cosine graphs affected by the coefficient of the functions and/or the coefficient of the angle?
73	A2.A71	Lesson #14 Aim: How do we sketch the graph of $y = \tan x$? Students will be able to 1. sketch the graph of $y = \tan x$ 2. use a graphing calculator to graph $y = \tan x$ 3. determine the period, domain and range of $y = \tan x$ 4. explain why $y = \tan x$ does not have an amplitude 5. explain how the tangent graph differs from the sine graph and the cosine graph 6. define *asymptote* and sketch the asymptotes of the graph of $y = \tan x$ 7. explain how the graph of $y = \tan x$ varies in each quadrant 8. compare the graph of y=tan x to the graphs of y = sin x and y = cos x Writing Exercise: The graph of $y = \tan x$ is a non-continuous periodic function. Explain the characteristic(s) of the tangent function that causes these discontinuities..
74	A2.A71	Lesson #15 Aim: How do we sketch the graphs of y = csc x, y = sec x, and y = cot x? Students will be able to: 1. sketch y = csc x, y = sec x, and y = cot x 2. use the graphing calculator to graph y = csc x, y = sec x, and y = cot x 3. compare and contrast the properties of the graphs of the 6 trig functions 4. write the equation of a trig function that is represented by a given periodic graph Writing Exercise: Ramon claims that even though y=csc x and y=sin x are reciprocal functions, the graph of y=csc x has more in common with the graph of y=tan x than it does with y=sin x. Evaluate his statements and give evidence to support your judgment.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
75	A2.A65	Lesson #16 Aim How do we sketch the graphs of the inverses of the sine, cosine, and tangent functions? Students will be able to: 1. define inverse of a function 2. apply the transformation $r_{y=x}$ to the sine, cosine, and tangent functions 3. apply the vertical line test to explain why the reflected graph is not a function 4. discover the restricted domain of the sine, cosine, and tangent graphs to ensure that the reflected graph is a function 5. state the domain and range of the inverse sine, cosine, and tangent functions 6. sketch the inverse sine, cosine, and tangent functions 7. use the calculator to graph the inverse sine, cosine, and tangent functions Writing Exercise: Explain why we sometimes have to restrict the domain of the inverse of a function.
76	A2.A63 A2.A64 A2.A65	Lesson #17 Aim: How do we evaluate inverse trigonometric relations and functions? Students will be able to 1. form the inverse of a given trigonometric function 2. transform between direct trigonometric notation and inverse trigonometric notation 3. evaluate expressions involving inverse trigonometric notation 4. state the principal range of the inverse trigonometric functions 5. use the calculator to evaluate expressions using principal value notation Writing Exercise: Explain why the inverse of the sine function is only a function in a restricted domain.
77	A2.A68	Lesson #18 Aim: How do we solve linear trigonometric equations? Aim Students will be able to 1. solve a linear trigonometric equation for the trigonometric function 2. find the reference angle based upon the value of the function 3. find all the solutions to a linear trigonometric equation given a specific domain Writing Exercise: How is the solution set of a linear equation different from the solution set of a linear trigonometric equation?
78	A2.A68	Lesson #19 Aim: How do we solve quadratic trigonometric equations? Students will be able to 1. solve quadratic trigonometric equations either by factoring or by using the quadratic formula 2. solve quadratic trigonometric equations for all values of the angles whose measures lie between $0°$ and $360°$ Writing Exercise: How would the solution set of $\sin^2\theta = 1$ for $0 \le \theta < 2\pi$ differ from the solution set for $\sin^2\theta = 1$ for an unrestricted domain?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
79	A2.A68	Lesson #20 Aim: How do we solve trigonometric equations that contain more than one function? Aim Students will be able to 1. apply previously learned identities to express an equation in terms of one trigonometric function 2. solve the resulting equation for all values of the angle in the interval $0^0 \leq \theta \leq 360^0$ Writing Exercise: A trigonometric equation that contains more that one function is like an equation with two variables. Compare and contrast the techniques that are used to solve equations with two variables with trigonometric equations that contain more than one function
80	A2.A74	Lesson #21 Aim: How do we find the area of a triangle given the lengths of two adjacent sides and the included angle? Student will be able to 1. investigate and discover the formula for the area of a triangle in terms of two sides and the sine of the included angle. 2. conjecture and apply the formula A = $\frac{1}{2}ab\sin C.$ to write a formula for the area of a parallelogram in terms of two sides and the sine of the included angle 3. apply either area formula to solve problems, including real-world applications involving triangles and parallelograms Writing Exercise: The area of a triangle can be determined using either of the following formulas: $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab\sin C.$ Explain how these two formulas are related.
81	A2.A73	Lesson #22 Aim: What is the Law of Sines? Students will be able to 1. investigate and discover the Law of Sines from the formula A = $\frac{1}{2}ab\sin C.$ 2. express the Law of Sines in different forms 3. explain the conditions necessary to apply the Law of Sines 4. apply the Law of Sines to find the length of a side of a triangle, if measures are given for two angles and a side (in short numerical problems only) 5. justify whether or not a triangle is acute, obtuse, or right Writing Exercise: Ptolemy was aware of the Law of Sines in the 2nd century B.C.. Use the Internet or your local library to find out how the Greeks used this theorem.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
82	A2.A73	Lesson #23 Aim: How do we apply the Law of Sines? Students will be able to 1. apply the Law of Sines to find the measure of a side or angle of a triangle 2. solve long problems involving the use of the Law of Sines 3. explain the conditions necessary to apply the Law of Sines 4. use the calculator to find the sine of an angle expressed in degrees with minutes or decimal Writing Exercise: The Law of Sines has led to a practice called “triangulation.” Find out what this practice is. Explain how it can be used by forest rangers to locate the position of a forest fire or how it can be used by the Coast Guard to find someone lost at sea.
83	A2.A73 A2.A75	Lesson #24 Aim: How can the Law of Sines be used in problems involving the “ambiguous case?” Students will be able to 1. apply the Law of Sines to discover all possible values of an unknown angle 2. conjecture and justify the number of possible triangles 3. explain the nature of all possible triangles Writing Exercise: When a Kodiak bear fishes for his breakfast in a river, the bear is aware that the fish is not located in the position in which he sees it. People understand that this is because a beam of light that strikes the surface of the river water is bent or refracted. This relationship between the speed of light (both in and out of the water) is described in Snell’s Law. How is Snell’s Law related to the Law of Sines?
84	A2.A73	Lesson #25 Aim: What is the Law of Cosines? Students will be able to 1. explore and discover the Law of Cosines 2. express the Law of Cosines in various ways 3. solve short problems using the Law of Cosines 4. compare and contrast the conditions necessary to use the Law of Cosines as opposed to the Law of Sines Writing Exercise: After using the Law of Sines to find a missing angle of a triangle, Barbara determined that $\sin A = 1.75$. What conclusion can you draw about the triangle?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
85	A2.A73	Lesson #26 Aim: How do we apply the Law of Cosines? Students will be able to 1. explain the circumstances necessary to apply the Law of Cosines 2. apply the Law of Cosines to solve triangle problems 3. solve problems involving angle measurement of a circle and the Law of Cosines 4. apply the Law of Cosines, given the lengths of the three sides of a triangle 5. justify whether or not a triangle is acute, obtuse, or right 6. (Honors Topic) apply the Law of Cosines to real-world problem involving the parallelogram of forces Writing Exercise: During a trial on a TV show, the witness' testimony indicated the relative positions of the defendant and the victim. His testimony also indicated his position as he witnessed the crime. The police investigation determined that the victim and the defendant would have to have been 30 feet apart while the distance of the witness to the victim would have to be 65 feet and the witness' distance to the defendant would have to be 25 feet. Describe how the Law of Cosines can be used to show that the testimony is flawed. Is there a simpler way to show that the testimony has errors?
86	A2.A73	Lesson #27 Aim: How do we determine the appropriate formulas to use in solving triangle problems? Students will be able to 1. explain when trigonometry of the right triangle is used to find lengths of sides or measures of angles 2. explain when the Law of Sines is used to find lengths of sides or measures of angles 3. explain when the Law of Cosines is used to find lengths of sides or measures of angles 4. solve problems involving any combination of the Law of Sines, Law of Cosines and trigonometry of the right triangle 5. solve numerical examples involving trigonometric ratios, including angle of elevation Writing Exercise: Often it is not possible to make measurements directly, as in the case of determining the elevation of a mountain peak. Describe how the Law of Sines and/or the Law of Cosines can help with such measurements.
87	A2.A76	Lesson #28 Aim: How do we find the cosine of the difference of two angles and the cosine of the sum of two angles? Students will be able to 1. verify the validity of the formula for the cosine of the difference of two angles 2. verify the validity of the formula for the cosine of the sum of two angles 3. apply the formulas for $\cos(A-B)$ and $\cos(A+B)$ to find the exact value of expressions involving angles measured in radians and in degrees 4. state the sum and difference formulas in words Writing Exercise: Explain why we cannot use the distributive law to evaluate *cos(A+B)* as *cosA+cosB*.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
88	A2.A76	Lesson #29 Aim: How do we find the sine of the difference of two angles and the sine of the sum of two angles? Students will be able to 1. apply the formulas for cos(A+B) and cos(A-B) to discover the formulas for $\sin(A-B)$ and $\sin(A+B)$ 2. verify the validity of the formula for the sine of the difference of two angles 3. verify the validity of the formula for the sine of the sum of two angles 4. apply the formulas for $\sin(A-B)$ and $\sin(A+B)$ to find the exact value of expressions involving angles measured in radians and in degrees 5. state the sum and difference formulas in words Writing Exercise: How can we use the formulas learned today to derive the formula for $\sin(A+B+C)$?
89	A2.A76	Lesson #30 Aim: How do we find the tangent of the sum of two angles and the tangent of the difference of two angles? Students will be able to 1. apply the sum and difference formulas for sine and cosine to discover formulas for the tangent of the sum of two angles and the tangent of the difference of two angles Writing Exercise: Explain how to derive the formula for $\cot(A+B)$.
90	A2.A77	Lesson #31 Aim: How do we find the value of trigonometric functions of double angles? Students will be able to 1. apply the formulas for sin(A+B), cos (A+B) and tan(A+B) to discover the formulas for the sin (2x), cos(2x) and tan(2x) 2. verify the validity of the formula for the sine, cosine and tangent of the angle 2x 3. apply the formulas for sin(2x), cos (2x) and tan(2x) to find the exact value of expressions involving angles measured in radians and in degrees 4. state the double angle formulas in words Writing Exercise: Explain why sin(2A) is not equal to 2sinA.
91	A2.A77	Lesson #32 Aim: How do we find the value of trigonometric functions of half angles? 1. discover the half-angle formulas 2. verify the validity of the formula for the sine, cosine and tangent of the angle ½ x 3. apply the formulas for sin(½ x), cos (½ x) and tan(½ x) to find the exact value of expressions involving angles measured in radians and in degrees 4. state the half angle formulas in words Writing Exercise: Explain how to determine the sign of the answer when using a half angle formula.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
92	A2.A68	Lesson #33 Aim: How do we apply the double angle formulas to solve trigonometric equations? Students will be able to 1. state the trigonometric formulas involving double angles 2. decide which double angle formula is needed to solve a trigonometric equation 3. solve trigonometric equations using double angle formulas 4. express solutions to the required degree of accuracy in the specified interval Writing Exercise: 1. Explain why is $\cos 2x = 1 - 2\sin^2 x$, a better choice to use than $\cos 2x = 2\cos^2 x - 1$ in order to solve $\cos 2x + \sin x + 3 = 0$. 2. Since there are three expansion formulas for y=cos 2θ, how we decide which formula substitution is best to use when solving a trig equation containing cos 2θ? Give one or more examples.
93	A2.A29 A2.A30 A2.A32	Lesson #34 Aim: How do we use an arithmetic sequence to solve problems? Students will be able to: 1. define what is meant by an arithmetic sequence and its common difference 2. determine whether a given sequence is an arithmetic sequence 3. determine the common difference, d, for the nth term of an arithmetic sequence 4. discover the formula for the nth term of an arithmetic sequence, $a_n = a_1 + (n-1)d$ 5. explain how to find a specified term of an arithmetic sequence 6. solve numeric, algebraic, and verbal problems using the relationship $a_n = a_1 + (n-1)d$ for a arithmetic sequence Writing exercise: How you can tell whether a sequence is an arithmetic sequence?
94	A2.A29 A2.A31 A2.A32	Lesson #35 Aim: How do we use a geometric sequence to solve problems? Students will be able to: 1. define what is meant by a geometric sequence and its common ratio. 2. determine whether a given sequence is a geometric sequence, an arithmetic sequence, or neither 3. determine the common ratio, r, for the nth term of a geometric sequence 4. discover the formula for the nth term of a geometric sequence, $a_n = a_1r^{n-1}$ 5. explain how to find a specified term of a geometric sequence 6. solve numeric, algebraic, and verbal problems using the relationship $a_n = a_1r^{n-1}$ for a geometric sequence Writing exercise: How you can tell whether a sequence is a geometric sequence? Compare and contrast an arithmetic sequence to a geometric sequence.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
95	A2.A35	Lesson # 36 Aim: How do we find the sum of the first n terms of an arithmetic series? Students will be able to: 1. define series and arithmetic series 2. compare and contrast an arithmetic sequence and an arithmetic series 3. explore and discover the formula $s_n = \frac{n}{2}(a_1 + a_n)$ for finding the sum of the first n terms of an arithmetic series 4. apply the formula $s_n = \frac{n}{2}(a_1 + a_n)$ to problems when given as a sum or described verbally Writing Exercise: Describe a method for quickly finding the sum of all the natural numbers from 1 to 100. Explain why your method works.
96	A2.A35	Lesson # 37 Aim: How do we determine the sum of the first n terms of a geometric series? Students will be able to: 1. define geometric series 2. compare and contrast a geometric sequence and a geometric series 3. explore and discover the formula $S_n = \frac{a_1 - a_1 r^n}{1-r}$ for finding the sum of n terms of a geometric series 4. apply the formula $S_n = \frac{a_1 - a_1 r^n}{1-r}$ to problems when given as a sum or described verbally Writing Exercise: Describe a real-world situation that involves finding the sum of a geometric series. Indicate what the common ratio is in your description.
97	A2.A34 A2.A35 A2.N10	Lesson # 38 Aim: How can we use summation notation to represent a series? Students will be able to: 1. define summand, limits of summation, and index 2. use summation notation to represent a series of n-terms for arithmetic, geometric, linear, quadratic, trigonometric, imaginary series 3. find the sum of n-terms of an arithmetic or geometric series given in summation notation Writing Exercise: Many letters of the Greek alphabet are used to represent mathematical ideas. List three such letters and describe the mathematical ideas they are used to represent.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
98	A2.A33	Lesson # 39 Aim: How do we specify the terms of a sequence by relating them to previous terms? Students will be able to: 1. define recursion 2. write the terms of a sequence given a recursive rule and the first term 3. write a recursive rule for a given sequence Writing Exercise: When should a recursive rule be used to indicate a sequence rather than an explicit rule in terms of n? Writing Project: How is recursion used to generate the Fibonacci numbers? What are the Fibonacci numbers? Describe their origin and occurrences in the real-world.
99	A2.S13 A2.S14	Lesson # 40 Aim: How do we compute theoretical, empirical and geometric probability? Students will be able to: 1. differentiate between empirical and theoretical probability 2. state the Counting Principle 3. compute theoretical probability using the Counting Principle 4. compute geometric probabilities such as finding the probability that a randomly selected point will lie inside of a geometric figure 5. perform experiments to approximate a probability empirically 6. use the graphing calculator's random number generator to simulate experiments 7. find the probability of a complement of an event Writing exercises: Explain how we could use geometric probability to approximate the value of π . Exploration in Probability: Use a random number generator to generate thirty random numbers that will simulate tossing a fair coin thirty times. Let an even number represent the result of the coin coming up heads, and an odd number represent the result of tails. Generate a second list of thirty random numbers to represent the second toss of a fair coin. Use the two lists of random numbers to compute the following empirical probabilities: P(head, tail), P(both the same), P(both different). Compute the following theoretical probabilities for the same experiment: P(head, tail), P(both the same), P(both different). How do the empirical and theoretical probabilities compare?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
100	A2.S12	Lesson # 41 Aim: How do we use the Fundamental Counting Principle to determine the number of elements in a sample space? Students will be able to: 1. state and apply the Counting Principle to specific problems 2. define independent event, dependent event 3. explain what is meant by sampling with and without replacement 4. apply the counting principal to find probabilities of compound events Writing exercise: When one event is made up of a series of choices, we can often make a tree diagram to illustrate all the possible ways the event can occur. How does the tree diagram support the principle that multiplication can be used to compute the total number ways the event can occur?
101	A2.S9 A2.S10	Lesson # 42 Aim: How do we solve problems using permutations? Students will be able to: 1. define permutation, factorial (!), ${}_nP_n = n!$, and ${}_nP_r$ 2. apply factorials to compute the number of arrangements of n different objects taken n at a time 3. compute the number of permutations of n things taken n at a time 4. employ the notation ${}_nP_r$ in solving problems involving n things taken r at a time 5. compute the number of permutations involving n things taken r at a time 6. discover a formula for the number of permutations of n objects with r of them identical 7. apply the counting principle along with permutations to count the elements in a sample space 8. use the calculator to compute permutations and factorials 9. use permutations with and without the counting principle to determine the probability of an event 10. use the calculator to compute permutations Writing Exercise: 1. Explain the meaning of a permutation. 2. Make up a problem in which the permutation formula would be used to solve it.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
102	A2.S9 A2.S11	Lesson # 43 Aim: How do we use combinations to solve probability problems? Students will be able to: 1. define a combination 2. discover a formula for the combination of n different objects taken r at a time 3. identify the notations used with combinations 4. explain the circumstances under which a permutation should be used or under which a combination should be used 5. apply the combination formula 6. apply the counting principle along with combinations to count the elements in a sample space 7. use the calculator to compute combinations 8. use combinations to solve probability problems Writing exercise: The lock on your gym locker is probably called a combination lock It needs a sequence of numbers rather than a key to open it. Is the word “combination” an appropriate description or would “permutation” lock be a more mathematically accurate name? Explain.
103	A2.S15	Lesson #44 Aim: How do we find the probability of a specific number of successes when an experiment is repeated n times? Students will be able to 1. explain what types of problems are Bernoulli experiments 2. discover the formula for computing exactly r successes in n independent trials 3. compute exactly r successes in n independent trials by using the Bernoulli formula Writing Exercise: Explain the necessary conditions for applying the Bernoulli formula to finding the probability of a particular event.
104	A2.S15	Lesson #45 Aim: How do we use Bernoulli’s Theorem to solve problems involving “at most” and “at least”? Students will be able to 1. investigate the meaning of *at least* and *at most* 2. express *at least* and *at most* Bernoulli experiments as the sum of the appropriate probabilities 3. solve Bernoulli problems involving *at least* and *at most* Writing Exercise: How are the phrases “at most three days” and “at least three days” different? How does this difference impact on a Bernoulli experiment?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
105	A2.S15 A2.A36	Lesson #46 Aim: What is meant by the Binomial Theorem? Students will be able to 1. construct the first n rows of Pascal's Triangle 2. apply Pascal's Triangle to determine the coefficients of a binomial expansion 3. discover the patterns in the expansion of $(x+y)^n$ 4. apply combinations to determine the coefficients of a binomial expansion 5. apply the Binomial Theorem to expand binomials Writing Exercise: How do the rows of Pascal's Triangle help in understanding the Binomial Theorem?
106	A2.A36	Lesson #47 Aim: How do we find a specific term of a binomial expansion? Students will be able to 1. determine the middle term of an expanded binomial by using the Binomial Theorem 2. determine a specific term of an expanded binomial by using the Binomial Theorem Writing Exercise: Explain what would happen if $(x+y)^{-3}$ were expanded using the Binomial Theorem.
107	A2.S1 A2.S2	Lesson # 48 Aim : How do we design an unbiased study? The students will be able to: 1. distinguish among the different kinds of studies (survey, observation, controlled experiment) 2. determine factors that may affect the outcome of each type of survey 3. from given descriptions of surveys explain why they fit the model of a specific type of study 4. explain the meaning of population and sample 5. tell whether a given method of data collection uses a population or a sample 6. given a variety of situations, determine which type of data collection should be implemented Writing exercises: 1. Suppose you were one of the students in charge of planning the senior trip. The choices are: a baseball fantasy camp, Colonial Williamsburg, or Disney World. For each of these choices, explain how you would design a study that would be BIASED so that it would lead people to believe that most of the seniors want to go on that particular trip. 2. A group of eight students decided that they wanted to lose weight. Four of them decided to walk a mile each school day before school. The other four of them decided to walk a mile each school day after school. All eight weigh themselves each Wednesday and report their weight to their math teacher, who is keeping it confidential. One student in the class says this is an experiment. A second student disagrees and says this is an observational study. A third student thinks this is just a survey. Write a paragraph to explain why you believe the study is an experiment, an observational study, or a survey. Be clear and concise

<table>
<tr><th>Lesson #</th><th>Performance Indicator</th><th>Aim and Lesson Performance Objectives</th></tr>
<tr><td>108</td><td>A2.S3</td><td>Lesson #49 Aim: How do we organize data using frequency tables, stem-and-leaf plots, and histograms?

Students will be able to
1. organize data in a frequency table
2. draw a histogram by hand for a given set of data
3. use the graphing calculator to draw a histogram for a given set of data
4. organize data in a stem and leaf plot
5. compare and contrast the advantages of organizing data in frequency tables, stem-and leaf-plots, and histograms
6. define mean, median, and mode
7. find the missing value in a data table given the mean, median and/or mode

Writing Exercises:

1. Data points in an experiment often repeat. Frequency tables allow us to organize the data into a compact table. To find the median score, the middle score must be located. How is it possible to use a frequency table to locate the median score?
2. The table displays the frequency of scores on a twenty point quiz. The mean of the quiz scores is 18.
<table>
<tr><td>Score</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>Frequency</td><td>2</td><td>4</td><td>7</td><td>13</td><td>k</td><td>5</td></tr>
</table>
Explain in writing how the table can be in conjunction with the mean score to compute the value of k shown in the table. How can the table be used to compute the median and the mode?</td></tr>
<tr><td>109</td><td>A2.S3</td><td>Lesson # 50 Aim: How do we apply measures of central tendency to solve problems?

The students will be able to:
1. explain what is meant by grouped data
2. define mean, median, and mode
3. calculate mean, median, and mode from a grouped frequency distribution chart
4. calculate mean, median, and mode from stem and leaf plot
5. use the calculator to compute mean, median, and mode
6. compute weighted averages
7. apply measures of central tendency to solve problems

Writing Exercise: Your test scores are 100,100,100,100,100,100,100,100,50, and 50. All were full period tests weighed equally. Your teacher claims that your average is 75 since your average on the first 8 exams was 100 and your average on the last 2 exams was 50. Explain how you would argue that this is incorrect.</td></tr>
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Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
110	A2.S4	Lesson # 51 Aim: How do we use measures of dispersion: range, variance, and standard deviation? The students will be able to: 1. define range, absolute deviation, variance and standard deviation 2. compare and contrast the meaning of sample and population 3. use a graphing calculator to calculate range, variance and standard deviation for both samples and populations 4. apply measures of dispersion to real-world problems 5. interpret meaning of measures of dispersion in real world situations Writing Exercise: The mean for a math test and for a science test were each 80. The standard deviation of the math test was three and of the science test was five. If Beverly scored an 87 on each test, in which class did she do better when compared to her classmates? Explain your answer.
111	A2.S4	Lesson # 52 Aim: How do we use measures of dispersion for grouped data? Students will be able to: 1. explain what is meant by grouped data, quartiles, and interquartile range 2. calculate quartiles and interquartile range, for both samples and populations, manually and using the graphing calculator 3. construct and interpret box and whisker plots 4. compute the standard deviation for grouped data of samples and populations 5. apply measures of dispersion in real world situations 6. find the missing value in a data table given a measure of central tendency and the standard deviation Writing exercise: The 5 numbers in the set {3, 4, 7, x, y} have a mean of 5 and a standard deviation of 2. If y > x, what is the value of x and y? Explain fully.
112	A2.S5	Lesson # 53 Aim : How do we apply the characteristics of a normal distribution? The students will be able to: 1. explain the properties of a normal distribution 2. apply the normal distribution to determine probabilities 3. determine what percent of normally distributed data is within a certain range given information about the mean and standard deviation 4. apply percentiles to the normal distribution 5. solve real world problems involving normal distributions Writing Exercise: How would an understanding of the normal distribution help the owner of a Big & Tall Men's Shoe store to stock the correct inventory of shoe sizes for his customers?

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
113	A2S5	Lesson # 54 Aim: How do z-scores help us to compare different data sets? The students will be able to: 1. define a z-score 2. compute the z-score for a data value 3. use the z-scores for individual data points to compare percentile ranks in different data sets 4. use z-scores in real world problems Writing Exercise: The SAT and the ACT are two different exams that students take when preparing to apply for colleges in the United States. The score range for these exams are very different. Describe how the admissions offices of a college can use the z-score is used to compare the academic standings of students who took the SAT to students who took the ACT.
114	A2.S16	Lesson # 55 Aim: How do we use the normal distribution as an approximation for binomial probabilities? The students will be able to: 1. interpret the area under a normal curve as a probability 2. interpret a binomial probability as a histogram and an approximation of the normal curve 3. find the mean and standard deviation of a binomial distribution 4. use the graphing calculator's normal cumulative density function feature (normalcdf) to approximate probabilities of Bernoulli trials involving "at least" and "at most." Writing Exercise: Give an example of an experiment where it is appropriate to use a normal distribution as an approximation for a binomial probability. Explain why in this example an approximation of the probability is a better approach than finding the exact probability.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
115	A2.S6 A2.S8	Lesson #56 Aim: How do we find the line of best fit for a set of data? Students will be able to 1. define and draw a scatter plot for a set of data 2. define regression, the least squares line and regression coefficients 3. state the properties of the least squares line 4. apply the properties of the least squares line to find the equation of the least squares line 5. define the correlation coefficient 6. apply the correlation coefficient to measure how closely the data points cluster about the least squares line 7. use the graphing calculator to find the equation of the least-squares line and the regression coefficients 8. apply the least squares line to real-world verbal problems Writing Exercises: 1. When the sales volume (in hundreds of units) is plotted along the x-axis, and the money spent on advertising (in thousands of dollars) is plotted along the y-axis, researchers obtained a least squares line with the equation: $y = 14x+0.7$. If $10,000 was spent in ads, do you have enough information to figure out the exact sales volume? Explain. 2. Why would you fit a linear model to data with a correlation coefficient that is close to zero?
116	A2S6	Lesson #57 Aim: How can we use the least-squares line to predict unknown values? Students will be able to 1. use the graphing calculator and charts of data to determine the equation for the least squares line 2. use the graphing calculator and problem generated data to determine the equation for the least squares line 3. use the equation found to predict the results for data points for which we do not have actual measurements 4. use the calculator generated correlation coefficient to determine how well the line fits the data Writing Exercise: Explain the significance of a negative correlation coefficient.

Lesson #	Performance Indicator	Aim and Lesson Performance Objectives
117	A2.S6	Lesson # 58 Aim: How do we determine from a scatter plot whether a linear, logarithmic, exponential, or power regression model is most appropriate? The students will be able to: 1. construct a scatter plot from a data set 2. sketch scatter plots whose regression models could be linear, logarithmic, exponential, or power regression models 3. explore the similarities and differences between and among the scatter plots of exponential, logarithmic, and power functions 4. determine from a given scatter plot or data set which model is most appropriate Writing Exercise: Properly computed regression models help scientists define relationships between two variables and use the resulting equation to predict future values. Scientists investigate possible correlations between different types of variables like: brain weight versus intelligence; gestation period verses life expectancy; attendance at math class verses final grade in the course. How do you think these variables are related? How could you use a scatter plot to decide the relationship between the data values for each of these studies?
118	A2.S7	Lesson # 59 Aim: How do we determine and utilize the regression function for a given set of data? The students will be able to: 1. use a scatter plot to help determine which regression model to use 2. use the graphing calculator to write the regression equation 3. use the regression function to make predictions based on the data 4. graph the regression function on the same set of axes as the scatter plot to determine how good a fit it is 5. use the regression function to interpolate and extrapolate from the data Writing Exercise: The cost of making a plain pizza of diameter x can be modeled by an equation of the form $ax^2 + k$ where k is a constant that accounts for fixed cost such as electricity for ovens rent and so forth. Explain why the term ax^2 makes sense.